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SOME NEW PROPOSED RIDGE PARAMETERS FOR THE LOGISTIC REGRESSION MODEL

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ABSTRACT

The parameter estimation method that based on the minimum residual sum of squares is unsatisfactory in the presence of multi collinearity. In (1970) Hoerl and Kennard introduced an alternative estimation approach which is called the ridge regression (RR) estimator. In RR approach, ridge parameter plays an important role in the parameter estimation. Many researchers are suggested various methods for determining the ridge parameter for the RR approach and the ygeneralized their methods to be applicable for the logistic ridge regression (LRR) model. Schaeffer et al. (1984) was the first who proposed a LRR estimator. In this article, new methods for choosing the ridge parameter for logistic regression (LR) are proposed. The performance of the proposed methods are evaluated and compared with other models that having different previously suggested ridge parameter through a simulation study in terms of mean square error (MSE). The developed technique in this communication seems to be very reasonable because of having smaller MSE. The results from the simulation study generally show that all the LRR estimators have alower MSE than the maximum like lihood(ML) estimator and our suggested LRR estimators were superior in most of the cases.

KEYWORDS: Logistic Regression, Maximum Likelihood, Monte Carlo Simulations, MSE, Multicollinearity, Ridge Regression, Ridge Parameter

INTRODUCTION

The concept of multi collinearity was first introduced by Frisch (1934), which occurs when the independent variables in a multiple regression model are collinear. This problem, which isvery common in applied researches, causes high variance and instable parameter estimates when estimating both linear regression models using ordinary least squares(OLS) technique and the LR model using the maximum likelihood estimation (MLE) method. There are several ways to solve this problem. One popular way to deal with this problem is called the ridge regression that first proposed by [Hoerl and Kennard (1970)]. The RR is known as an efficient remedial measure for the linear regression model and the LR model. A lot of researches mainly focused on different ways of estimating the ridge parameter. The authors proved that there is a non-zero value of such ridge parameter for which the MSE for the coefficients β using the RR is smaller than theMSE of the OLS estimator or the ML estimator of the respective parameter. Many authors have worked with this area of research and developed and proposed different estimators for the RR parameter. To mention a few, Hoerl and Kennard (1970a), Hoerl et al. (1975),McDonald and Galarneau (1975), Lawless and Wang (1976),Schaeffer et al. (1984),Khalaf and Shukur (2005), Alkhamisi et al. (2006), and Muniz and Kibria (2009).

The main goal of this paper is to suggest some new methods for choosing the ridge parameter *k* for LR. The performance of these proposed methods is evaluated by comparing them with other previously suggested models that having different ridge parameter based on a simulation study in terms of MSE. Very promising results for our suggested methods are shown.

METHODOLOGY

In this section we propose some LRR estimators for estimating the ridge parameter ***k*** based on the work of Hoerl, Kennard and Baldwin in (1975), Schaefer et al. in (1948) and Dorugadein (2010).

Model and Estimation

Logistic regression is a widely used statistical method, the i[th] value ofthe vector of the response variable$\boldsymbol{Y_{n\times 1}}$of the regression model is Bernoulli distributed with the following parameter value:

$$\pi_i(x_i) = \frac{e^{x_i{'}\beta}}{1 + e^{x_i{'}\beta}} \qquad (1)$$

Where $x_i = [1, x_{i1}, x_{i2}, x_{i3}, \ldots x_{ip}]'$is the i[th] row of data matrix $X_{n\times(p+1)}$ which is a vector of *p* independent variables and constant,$\beta = (\beta_0, \beta_1 \beta_2 \ldots, \beta_p)'$ is (p+1)×1vector of the coefficients (unknown parameters), n is the sample size.

The most common method of estimating β is to apply the maximum likelihood estimation (MLE) approach, the ML estimator of β is given by:

$$\hat{\beta}_{MLE} = [X'WX]^{-1}X'W\,Z, \qquad (2)$$

Where W is a square matrix of order n with element$\pi_i(1-\pi_i)$,Z is an n×1 column vector with the i[th] elements: $z_i = \text{logit}[\hat{\pi}_i(x_i)] + \frac{y_i - \hat{\pi}_i}{\hat{\pi}_i(1-\hat{\pi}_i)}$. The asymptotic covariance matrix of the ML estimator equals:

$$\begin{aligned} Var(\hat{\beta}_{MLE}) = cov(\hat{\beta}_{MLE}) &= (X'WX)^{-1} \\ &= \{X' \text{diag}\,[\hat{\pi}_i(1-\hat{\pi}_i)]\,X\}^{-1} \end{aligned} \qquad (3)$$

The MSE of the asymptotically unbiased $\hat{\beta}_{MLE}$is:

$$\begin{aligned} MSE &= E(\hat{\beta}_{MLE} - \beta)'(\hat{\beta}_{MLE} - \beta) \\ &= Tr[Var(\hat{\beta}_{MLE})] = \sum_{j=1}^{p} \frac{1}{\lambda_j} \end{aligned} \qquad (4)$$

Where λ_j is the j[th] eigenvalue of the X'WX matrix. One of the drawbacks of using the MLE approach is that the MSE of the estimator becomes inflated when the independent variables are highly correlated because some of the eigen values will be small. As a remedy to this problem, caused by the multicollinearity, Schaefer et al. (1984) proposed the following LRR estimator.

$$\widehat{\beta_k} = (X'WX + kI_p)^{-1}X'WX\hat{\beta}_{MLE} \quad (5)$$

The MSE of the LRR estimator equals:

$$MSE = E(\hat{\beta}_k - \beta)(\hat{\beta}_k - \beta)'$$

$$= \sum_{j=1}^{p} \frac{\lambda_j}{(\lambda_j + k)^2} + k^2 \sum_{j=1}^{p} \frac{\beta_j^{\,2}}{(\lambda_j + k)^2} \quad (6)$$

There are several researcher mainly focused on different ways of estimating the ridge parameter ***k*** [1][13].

The Ridge Parameter

Estimating the value of the ridge parameter ***k*** is an important problem in the RR method. Many different techniques for estimating k have been proposed by various researchers. The RR estimator does not provide a unique solution to the problem of multi collinearity but provides a family of solutions, because here is no specific rule for how to choose the ridge parameter. These solutions depend on the value of ***K*** which is the diagonal matrix of the non-negative constants k_j. A useful procedure uses $K = kI$, k>0. However, several methods have been proposed for the linear RR model, and these methods have been generalized to be applicable for the LRR model. The most classical RR parameters are summarized in Table 1.

Table1: Some Common RR Parameters

Author	Ridge parameter
Proposed by Hoerl and Kennard (1970),	$k_j = \frac{\hat{\sigma}^2}{\hat{\beta}_j^2} \quad j = 1,2,\dots p$
Proposed by Hoerl and Kennard (1970)	$k_{HK} = \frac{\hat{\sigma}^2}{\hat{\beta}^2_{max}},$
Suggested byHoerl, Kennard and Baldwin (1975)[4]	$k_{HKB} = \frac{p\hat{\sigma}^2}{\hat{\beta}'\hat{\beta}} = \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p}\beta_i^2}$
Proposed by Schaefer et al. (1984)	$k_{SRW} = \frac{1}{\hat{\beta}^2_{max}}$
Suggested by Khalaf and Shukur (2005)	$k_{ks} = \frac{\hat{\sigma}^2\lambda_{max}}{(n-p-1)\hat{\sigma}^2 + \lambda_{max}\hat{\beta}^2_{max}}$
Suggested by Dorugade and Kashid (2010)	$k_D = \max\left(0, \frac{p\hat{\sigma}^2}{\hat{\alpha}'\hat{\alpha}} - \frac{1}{n(VIF_j)_{max}}\right)$

Where$\hat{\sigma}^2$ is the residual variance of the raw residuals divided by the degrees of freedoms (n − p −1), λ_{max} is the largest eigenvalue of the matrix $X'X$ and $VIF_j = \frac{1}{1-R_j^2}$is the variance inflation factor of the j^{th}regressor.

New Proposed Ridge Parameter

In this section, three different methods of specifying the ridge parameter $\boldsymbol{k}$ will be proposed. Those three methods are considered to be a modification of three others ridge parameters proposed elsewhere. Our main goal is to give three new estimators with smaller MSE value compared with other previously suggested ridge estimators. The first new proposed ridge parameter, $\boldsymbol{k_{SA1}}$, and hence its estimator is a modification of the estimator which is proposed by [Dorugade (2010)]. The mathematical formula of $\boldsymbol{k_{SA1}}$ is as follows:

$$k_{SA1} = \max\left(0, \frac{p\hat{\sigma}^2}{\hat{\hat{\beta}}'\hat{\hat{\beta}}} - \left[\frac{1}{n(VIF_j)_{max}}\right]^2\right), \tag{7}$$

Where$\hat{\beta}$ is the ML estimator of β.By squaring the term $\left[\frac{1}{n(VIF_j)_{max}}\right]$, the value of ridge parameter will be increased, and as a consequence the bias of theproposed estimator will be also increased, and this will reduce the MSE ofthe corresponding ridge estimator.

The second and the third modified ridge parameters are given by the following formulas:

$$k_{SA2} = \frac{p\hat{\sigma}^2}{\hat{\hat{\beta}}'\hat{\hat{\beta}}} * \left[\frac{n}{p(VIF_j)_{max}}\right]^{\frac{1}{p}} \tag{8}$$

$$k_{SA3} = \frac{1}{\hat{\beta}^2{}_{max}} * \left[\frac{n}{p(VIF_j)_{max}}\right]^{\frac{1}{p}} \tag{9}$$

The $\boldsymbol{k_{SA2}}$ ridge parameter is an enhancement of the ridge parameter which is given by [Hoerl, Kennard and Baldwin] in (1975). While $\boldsymbol{k_{SA3}}$ is a modification of the ridge parameter which is suggested by [Schaefer et al.] in (1948).

Our goal is to multiply those two previously suggested ridge parameters by the term$\left[\frac{n}{p(VIF_j)_{max}}\right]^{\frac{1}{p}}$, which is often greater than one. So the value of the bias of the two new suggested estimators will be increased, and this will give an opportunity for a large reduction of the MSE criterion of the two new suggested ridge estimators.

Simulation Study

In this section, the performance of the three suggested ridge estimators is evaluated over several different ridge estimators. Since a theoretical comparison is not possible, a simulation study is conducted in this section. The design of a good simulation study is depended on:

- What factors are expected to affect the properties of the estimators under investigation, and
- What criteria are being used to judge the results.

Factors Affecting the Properties of Estimators

In this section, a brief description of the selected factors that is used in the simulation study with different values will be presented.

- **The Strength of Correlation among the Predictor Variables (ρ^2)**

The most obvious factor that affects the properties of the different estimators is the degree of correlation between the independent variables. The four different degrees of correlation that are used in this simulation study are:

$\rho^2 = 0.70, 0.80, 0.90$ and 0.95.

- **The Number of Independent Variables(P)**

Another factor that has an obvious effect on the evaluation of the estimators is the number of independent variables. The main interest of varying this factor is to see which ridge parameter is the best for specific number of independent variables. In most simulation studies the proposed ridge estimator is calculated using a fairly low number of predictor variables (2 and 4 is the most common selected value of p)[14]. Hence, there is a need to conduct an investigation where more variables are considered to see the effect of increasing the number of independent variables on the performance of the ridge estimators. The number of independent variables that is used in the simulated models is equal to 2, 3, 4, 5, 10.

- **The Sample Size (n)**

Another consideration that is taken into account is the sample size n.Actually, when comparing different estimation methods, increasing the n is supposed to have a positive effect on the MSE, as increasing the n leads to a lower variance of the estimated parameters. Therefore, it is interesting to investigate the gain of using LRR when n is both small and large. The sample size is increased with the number of independent variables (p). Many papers show that to obtain meaningful results from the LR model, the sample size is needed to be adjusted. Therefore, the number of observations that are used in this simulation study is depend on 20p+10, 30p, 40p, 60p, and 100p, respectively [13][17].

Criteria for Measuring the Goodness of an Estimator

The MSE is used as a criterion to measure the goodness of the estimator. It is used to compare the new three proposed ridge estimators with other four previously suggested ridge estimators together with the ML estimator. For a given values of *p*, *n*, and ρ^2 the set of predictor variables are generated. Then the experiment was repeated 1,000 times by generating new error terms. After that the values of the ML estimator, also the previously suggested and the modified ridge parameters *k* and their corresponding ridge estimators as well as the average MSE (AMSEs) are evaluated for each estimator.

Generation of Independent and Dependent Variables

Following Gibbons (1981), and to achieve different degrees of collinearity, the predictor variables are generated using the following equation:

$$x_{ij} = (1-\rho^2)^{\left(\frac{1}{2}\right)} z_{ij} + \rho z_{ip}, \quad \text{where} \qquad (10)$$

$i = 1,2,\dots,n$, $j = 1,2,\dots,p$, ρ^2represents the correlation between any two predictor variables and $z_{ij}$are independent standard normal pseudo-random numbers. The n observations for the dependent variable are obtained from the

Bernoulli (π_i) distribution in Equation (1). The values of the parameters $\beta_1, \beta_2, \beta_3, \ldots, \beta_p$ are chosen so that $\beta_1 = \beta_2 = \cdots = \beta_p$ and $\sum_{j=1}^{p} \beta_j = 1$, which is common restrictions in many simulation studies; [12].The value of the intercept is another important factor since it equals the average value of the log odds ratio. Hence, when the intercept equals zero then there is an equal average probability of obtaining one and zero. While, when the intercept is positive then the average value of the log odds ratio is positive which means that there is a greater probability of obtaining one than zero. Finally, when the value of the intercept is negative the opposite situation occurs which means that there is a greater probability of obtaining zero than one. Accordingly, the value of the intercept in the simulation study is chosen to be zero. [13]

RESULTS AND DISCUSSIONS

In this section, the main results of the Monte Carlo simulation concerning the properties of the estimation method for choosing the ridge parameter have been presented. The results of the simulated AMSEs are summarized in Tables [2-6] and Figures [1-10]. Those Tables and Figures show the effects of changing the sample sizes and the correlation coefficient values between the independent variables on the performance of ML and different ridge estimators.

According to our simulation study many conclusion can be drawn on the performance of the ML and different ridge modified and previously suggested estimators, these conclusion, can be summarized as follows:

- Almost all the cases indicates that the ML estimator performs worse than the modified and previously suggested ridge estimators except when (n=200 and ρ=0.7) the performance of estimator based on $\boldsymbol{k_{SA3}}$ was not good.
- Our first modified ridge estimator based on $\boldsymbol{k_{SA1}}$ perform better than $\boldsymbol{k_D}$ estimator in all cases.
- The second suggested ridge estimator based on $\boldsymbol{k_{SA2}}$ as a modification of $\boldsymbol{k_{HKB}}$ is also performs better than estimator based on $\boldsymbol{k_{HKB}}$ in most cases.
- Also $\boldsymbol{k_{SA3}}$ gives much better prediction results comparable with the ML estimator and the other modified and previously suggested estimators, this estimator seems to be superior at most of the cases.
- The estimator based on $\boldsymbol{k_{SA2}}$ is better than the estimator based on $\boldsymbol{k_{SA1}}$ when the correlation is not too high, but with the strong correlation the estimator based on $\boldsymbol{k_{SA1}}$ becomesbetter than the estimator based on $\boldsymbol{k_{SA2}}$. With increasing the sample size, the estimators based on $\boldsymbol{k_{SA1}}, \boldsymbol{k_{SA2}}$ are approaching to each other and the difference between them becomes small. The generally, estimator based on $\boldsymbol{k_{SA2}}$ is best in most cases.
- The ridge estimators that based on the parameters $\boldsymbol{k_{SA1}}$ and $\boldsymbol{k_{HKB}}$, have approximately the same results in most of the cases. The reason behind that is the $\boldsymbol{k_{SA1}}$ is a modification of the $\boldsymbol{k_D}$ parameter which, in the origin, is a modification of the $\boldsymbol{k_{HKB}}$ parameter. More specifically, when squaring the term $\left[\frac{1}{n(\mathrm{VIF_j})_{max}}\right]$, that is included in $\boldsymbol{k_{HKB}}$ its value approaches to zero and the value of $\boldsymbol{k_{SA1}}$ and the value of $\boldsymbol{k_{HKB}}$ become the same.

CONCLUSIONS

The performance of the three new proposed ridge estimators based on $\boldsymbol{k_{SA1}}, \boldsymbol{k_{SA2}}$ and $\boldsymbol{k_{SA3}}$ are shown to be better than the ML estimator in most of the cases. Our three suggested modifications give better prediction results than the previously suggested ridge estimators in most of the cases. Our third suggested ridge estimator that based on the ridge estimator which is proposed by [Schaefer et al. (1984)] looks superior to all the studied ML and ridge estimators as it has smaller AMSE in most of the cases.

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APPENDICES

Table 2: The AMSE of the ML and Different Ridge Estimators, For P=2 and Different Correlation and Sample Size

ρ^2	N	MLE	k_{HK}	k_{HKB}	k_{SBW}	k_D	k_{SA1}	k_{SA2}	k_{SA3}
0.7	50	0.8181	0.7133	0.6690 8	0.5340 6	0.6733	0.6691 1	0.5642	0.5361
0.80		1.2106	1.0039	0.9188	0.6970	0.9248	0.9189	0.7756	0.6650
0.90		2.4223	1.8110	1.5835	1.1182	1.5942	1.5835	1.4031	1.0513
0.95		4.9138	3.2621	2.7705	1.8680	2.7898	2.7706	2.7045	1.8408
0.7	60	0.6619	0.5856	0.5543	0.4462	0.5566	0.5543	0.4656	0.4617
0.80		0.9819	0.8291	0.7661	0.5854	0.7694	0.7661	0.6395	0.5712
0.90		1.9635	1.4983	1.3246	0.9386	1.3305	1.3247	1.1467	0.8808
0.95		3.9671	2.6887	2.2783	1.5345	2.2886	2.2783	2.1565	1.4877
0.7	80	0.4648	0.4195	0.4014	0.3321	0.4022	0.4014	0.3374	0.3687
0.80		0.6921	0.5980	0.5603	0.4399	0.5615	0.5603	0.4622	0.4499
0.90		1.3863	1.0880	0.9717	0.6997	0.9738	0.9717	0.8084	0.6601
0.95		2.8220	1.9672	1.6829	1.1460	1.6867	1.6829	1.5109	1.0870
0.7	120	0.2925	0.2712	0.2629	0.2259	0.2631	0.2629	0.2251	0.2776
0.80		0.4369	0.3922	0.3736	0.3061	0.3739	0.3736	0.3098	0.3379
0.90		0.8767	0.7257	0.6627	0.4985	0.6633	0.6627	0.5378	0.4888
0.95		1.7867	1.3215	1.1505	0.8061	1.1516	1.1505	0.9733	0.7558
0.7	200	0.1695	0.1613	0.1584	0.1419	0.1584	0.1584	0.1402	0.2032
0.80		0.2526	0.2344	0.2275	0.1951	0.2276	0.2275	0.1930	0.2447
0.90		0.5086	0.4439	0.4172	0.3282	0.4174	0.4172	0.3354	0.3433
0.95		1.0359	0.8213	0.7359	0.5318	0.7361	0.7359	0.5888	0.5045

Table 3: The AMSE of the ML and Different Ridge Estimators, for P=3 and Different Correlation and Sample Size

ρ^2	N	MLE	k_{HK}	k_{HKB}	k_{SBW}	k_D	k_{SA1}	k_{SA2}	k_{SA3}
0.7	70	1.1095	0.9862	0.9006	0.7162	0.9025	0.9006	0.8293	0.6683
0.80		1.7151	1.4736	1.3073	1.0024	1.3102	1.3073	1.2257	0.9461
0.90		3.5942	2.8829	2.4511	1.7856	2.4564	2.4511	2.4193	1.7635
0.95		7.4426	5.5414	4.5896	3.2725	4.5995	4.5896	4.8412	3.4348
0.7	90	0.8163	0.7342	0.6779	0.5501	0.6787	0.6779	0.6191	0.5159
0.80		1.2672	1.0994	0.9866	0.7674	0.9878	0.9866	0.9112	0.7201
0.90		2.6577	2.1570	1.8448	1.3644	1.8470	1.8448	1.7810	1.3239
0.95		5.5317	4.1948	3.4649	2.5019	3.4689	3.4649	3.5693	2.5660
0.7	120	0.5873	0.5381	0.5043	0.4176	0.5047	0.5043	0.4587	0.3936
0.80		0.9109	0.8105	0.7392	0.5873	0.7397	0.7392	0.6752	0.5476
0.90		1.9105	1.5893	1.3799	1.0285	1.3807	1.3799	1.3012	0.9798
0.95		3.9704	3.0618	2.5431	1.8301	2.5447	2.5431	2.5448	1.8315
0.7	180	0.3743	0.3499	0.3345	0.2863	0.3346	0.3345	0.3048	0.2766
0.80		0.5796	0.5281	0.4934	0.4044	0.4936	0.4934	0.4473	0.3797
0.90		1.2155	1.0450	0.9293	0.7108	0.9295	0.9293	0.8539	0.6638
0.95		2.5322	2.0292	1.7111	1.2469	1.7115	1.7111	1.6468	1.2074
0.7	300	0.2175	0.2075	0.2019	0.1798	0.2019	0.2019	0.1859	0.1818
0.80		0.3366	0.3147	0.3010	0.2573	0.3010	0.3010	0.2735	0.2493
0.90		0.7057	0.6299	0.5774	0.4600	0.5775	0.5774	0.5210	0.4286
0.95		1.4681]	1.2285	1.0666	0.8016	1.0663	1.0662	0.9853	0.7534

Table 4: The AMSE of the ML and Different Ridge Estimators, For P=4 and Different Correlation and Sample Size

ρ^2	N	MLE	k_{HK}	k_{HKB}	k_{SBW}	k_D	k_{SA1}	k_{SA2}	k_{SA3}
0.7	90	1.3154	1.1856	1.0662	0.8625	1.0672	1.0668	1.0217	0.8240
0.80		2.0812	1.8253	1.5951	1.2526	1.5967	1.5951	1.5556	1.2199
0.90		4.4369	3.6987	3.1023	2.3505	3.1053	3.1023	3.1544	2.3897
0.95		9.3517	7.4423	6.1193	4.5684	6.1253	6.1193	6.5239	4.8582
0.7	120	0.9293	0.8485	0.7743	0.6352	0.7747	0.7743	0.7356	0.6012
0.80		1.4643	1.3010	1.1506	0.9103	1.1512	1.1506	1.1069	0.8728
0.90		3.1266	2.6393	2.2224	1.6894	2.2235	2.2224	2.2184	1.6858
0.95		6.5701	5.2548	4.2960	3.1915	4.2982	4.2960	4.4993	3.3379
0.7	160	0.6704	0.6215	0.5769	0.4839	0.5771	0.5769	0.5460	0.4567
0.80		1.0555	0.9536	0.8600	0.6942	0.8602	0.8600	0.8196	0.6594
0.90		2.2559	1.9320	1.6551	1.2715	1.6556	1.6551	1.6241	1.2470
0.95		4.7336	3.8324	3.1498	2.3590	3.1507	3.1498	3.2339	2.4190
0.7	240	0.4294	0.4040	0.3825	0.3298	0.3825	0.3825	0.3613	0.3131
0.80		0.6763	0.6216	0.5736	0.4748	0.5737	0.5736	0.5419	0.4492
0.90		1.445	1.2695	1.1079	0.8691	1.1080	1.1079	1.0649	0.8348
0.95		3.0323	2.5078	2.0796	1.5707	2.0798	2.0796	2.0769	1.5685
0.7	400	0.2495	0.2387	0.2306	0.2051	0.2306	0.2306	0.2187	0.1978
0.80		0.3930	0.3692	0.3495	0.2994	0.3495	0.3495	0.3295	0.2844
0.90		0.8388	0.7559	0.6831	0.5498	0.6831	0.6831	0.6457	0.5210
0.95		1.7644	1.5117	1.2902	0.9962	1.2903	1.2902	1.2501	0.9669

Table 5: The AMSE of the ML and Different Ridge Estimators, For P=5 and Different Correlation and Sample Size

ρ^2	N	MLE	k_{HK}	k_{HKB}	k_{SBW}	k_D	k_{SA1}	k_{SA2}	k_{SA3}
0.7	110	1.4588	1.3227	1.1736	0.9564	1.1742	1.1736	1.1451	0.9287
0.80		2.3299	2.0606	1.7771	1.4075	1.7781	1.7771	1.7624	1.3940
0.90		5.0403	4.2767	3.5383	2.7321	3.5402	3.5383	3.6349	2.8094
0.95		10.6464	8.6723	7.0239	5.3775	7.0277	7.0239	7.4950	5.7297
0.7	150	1.0081	0.9274	0.8395	0.6981	0.8398	0.8395	0.8130	0.6720
0.80		1.6055	1.4421	1.2652	1.0200	1.2655	1.2652	1.2398	0.9965
0.90		3.4581	2.9697	2.4812	1.9340	2.4818	2.4812	2.5080	1.9564
0.95		7.3200	5.9994	4.8627	3.7198	4.8639	4.8627	5.1057	3.9057
0.7	200	0.7365	0.6861	0.6319	0.5327	0.6320	0.6319	0.6094	0.5102
0.80		1.1745	1.0694	0.9559	0.7777	0.9561	0.9559	0.9293	0.7524
0.90		2.5362	2.2140	1.8773	1.4710	1.8776	1.8773	1.8731	1.4670
0.95		5.3682	4.4652	3.6522	2.7984	3.6527	3.6522	3.7776	2.8930
0.7	300	0.4713	0.4445	0.4180	0.3598	0.4180	0.4180	0.4019	0.3439
0.80		0.7515	0.6943	0.6350	0.5273	0.6350	0.6350	0.6122	0.5057
0.90		1.6239	1.4385	1.2463	0.9894	1.2464	1.2463	1.2230	0.9696
0.95		3.4379	2.9092	2.4019	1.8656	2.4021	2.4019	2.4330	1.8892
0.7	500	0.2758	0.2644	0.2542	0.2259	0.2542	0.2542	0.2449	0.2170
0.80		0.4384	0.4133	0.3884	0.3328	0.3884	0.3884	0.3731	0.3178
0.90		0.9448	0.8560	0.7641	0.6188	0.7641	0.7641	0.7382	0.5959
0.95		1.9990	1.7278	1.4597	1.1425	1.4597	1.4597	1.4432	1.1295

Table 6: The AMSE of the ML and Different Ridge Estimators, For P=10 and Different Correlation and Sample Size

ρ^2	N	MLE	k_{HK}	k_{HKB}	k_{SBW}	k_D	k_{SA1}	k_{SA2}	k_{SA3}
0.7	210	2.0428	1.8974	1.6529	1.3933	1.6531	1.6529	1.6555	1.3960
0.80		3.3623	3.0749	2.6098	2.1720	2.6100	2.6098	2.6382	2.1994
0.90		7.4938	6.6820	5.4687	4.5201	5.4692	5.4687	5.6212	4.6534
0.95		16.1358	14.0717	11.2645	9.3006	11.2656	11.2645	11.7697	9.7128
0.7	300	1.3410	1.2554	1.1131	0.9457	1.1131	1.1131	1.1087	0.94091
0.80		2.1956	2.0215	1.7356	1.4561	1.7356	1.7356	1.7428	1.4632
0.90		4.8666	4.3506	3.5743	2.9680	3.5744	3.5743	3.6473	3.0317
0.95		10.4399	9.0865	7.2557	6.0123	7.2560	7.2557	7.5294	6.2332
0.7	400	0.9733	0.9174	0.8258	0.7059	0.8258	0.8258	0.8199	0.6988
0.80		1.5917	1.4744	1.2849	1.0782	1.2849	1.2849	1.2839	1.0772
0.90		3.5286	3.1717	2.6297	2.1842	2.6298	2.6297	2.6673	2.2171
0.95		7.5740	6.6173	5.3002	4.3918	5.3003	5.3002	5.4662	4.5264
0.7	600	0.6248	0.5934	0.5461	0.4695	0.5461	0.5461	0.5400	0.4621
0.80		1.0220	0.9543	0.8485	0.7122	0.8485	0.8485	0.8427	0.7061
0.90		2.2629	2.0493	1.7238	1.4233	1.7238	1.7238	1.7343	1.4328
0.95		4.8502	4.2590	3.4317	2.8279	3.4318	3.4317	3.5087	2.8910
0.7	1000	0.3667	0.3530	0.3339	0.2945	0.3339	0.3339	0.3297	0.2890
0.80		0.5988	0.5683	0.5219	0.4482	0.5219	0.5219	0.5161	0.4414
0.90		1.3243	1.2182	1.0551	0.8761	1.0551	1.0551	1.0518	0.8730
0.95		2.8409	2.5332	2.0828	1.7203	2.0828	2.0828	2.1055	1.7394

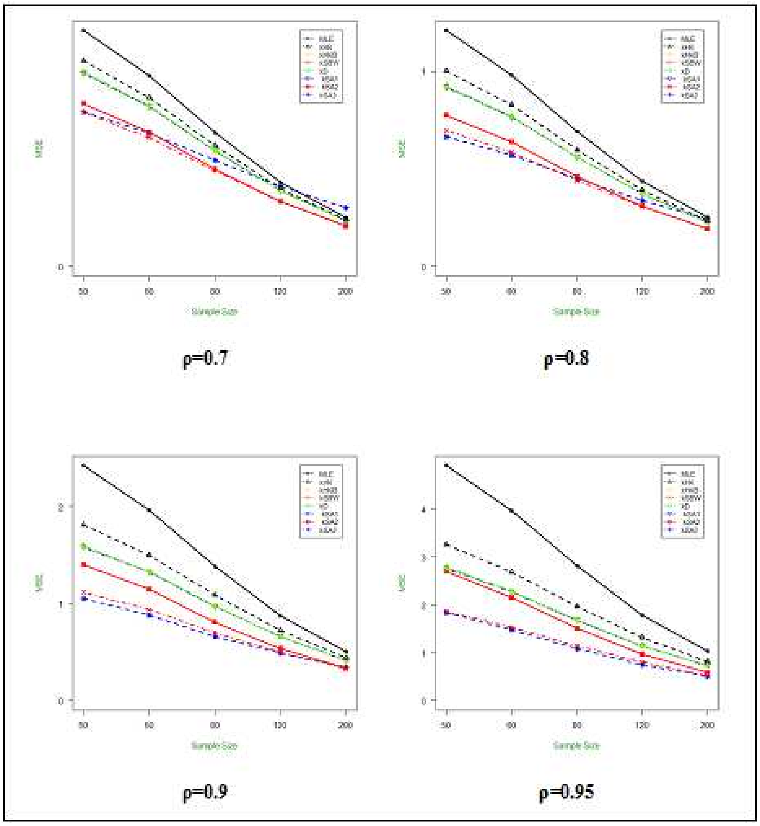

Figure 1: The AMSE of the Ml and Different Ridge Estimators, For P=2, P=0.70, 0.80, 0.90 and 0.95 with Different Sample Size

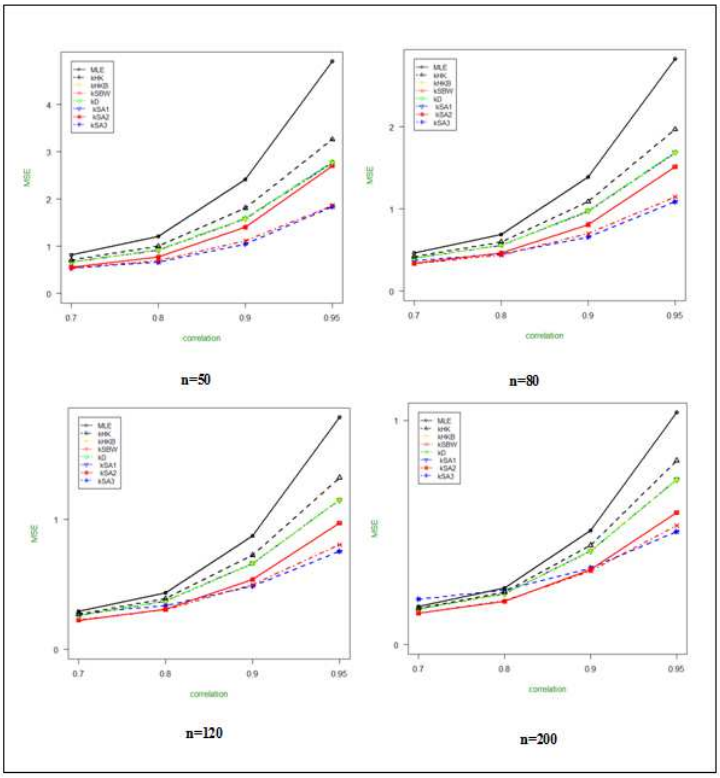

Figure 2: The AMSE of the ML and Different Ridge Estimators, For p=2, n=50, 80, 120 and 200 with Different Correlation

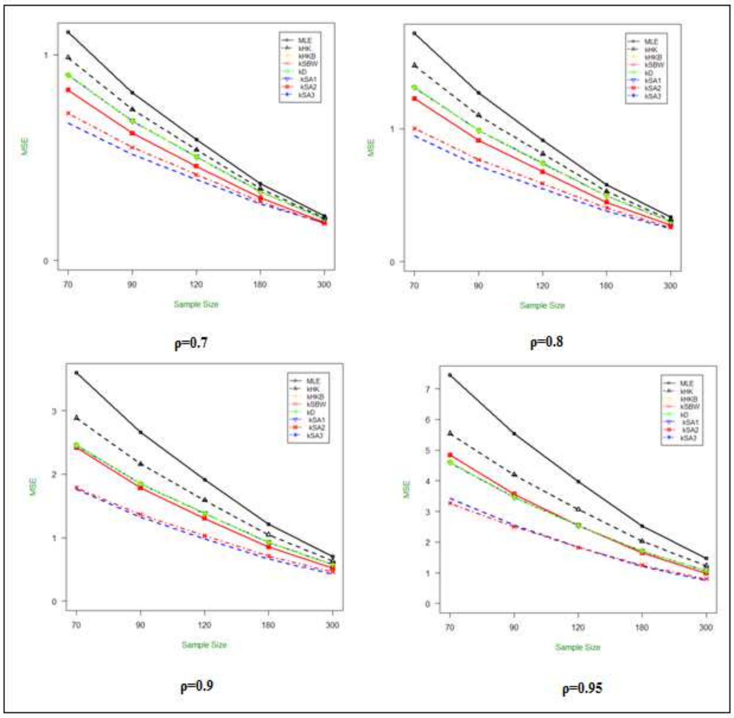

Figure 3: The AMSE of the ML and Different Ridge Estimators, For P=3, P=0.70, 0.80, 0.90 and 0.95 with Different Sample Size

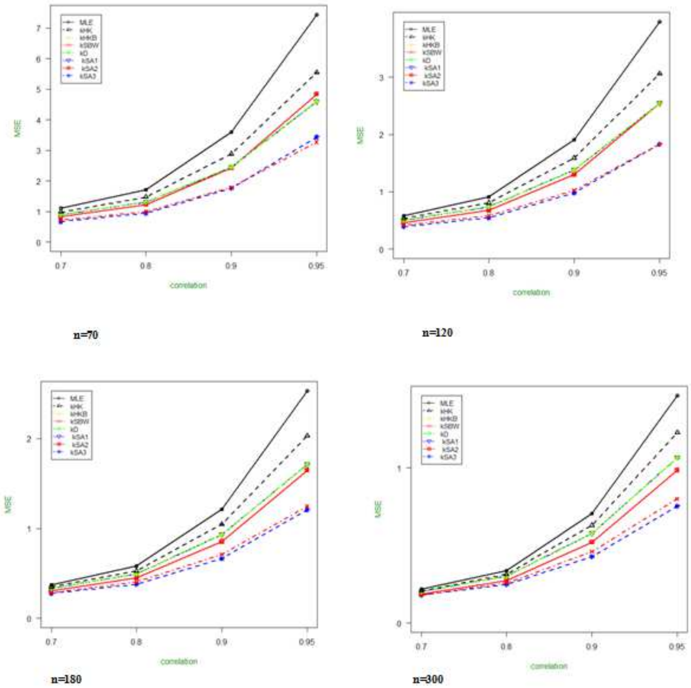

Figure 4: The AMSE of the ML and Different Ridge Estimators, For P=3, N=70, 120, 180 and 300 with Different Correlation

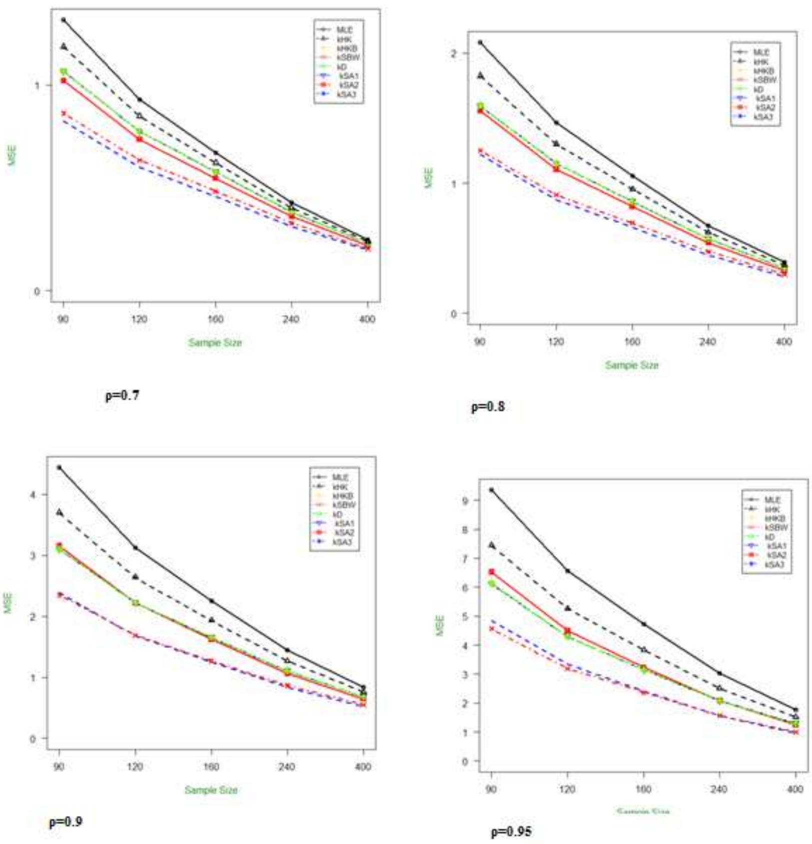

Figure 5: The AMSE of the ML and Different Ridge Estimators, For P=4, P=0.70, 0.80, 0.90 and 0.95 with Different Sample Size

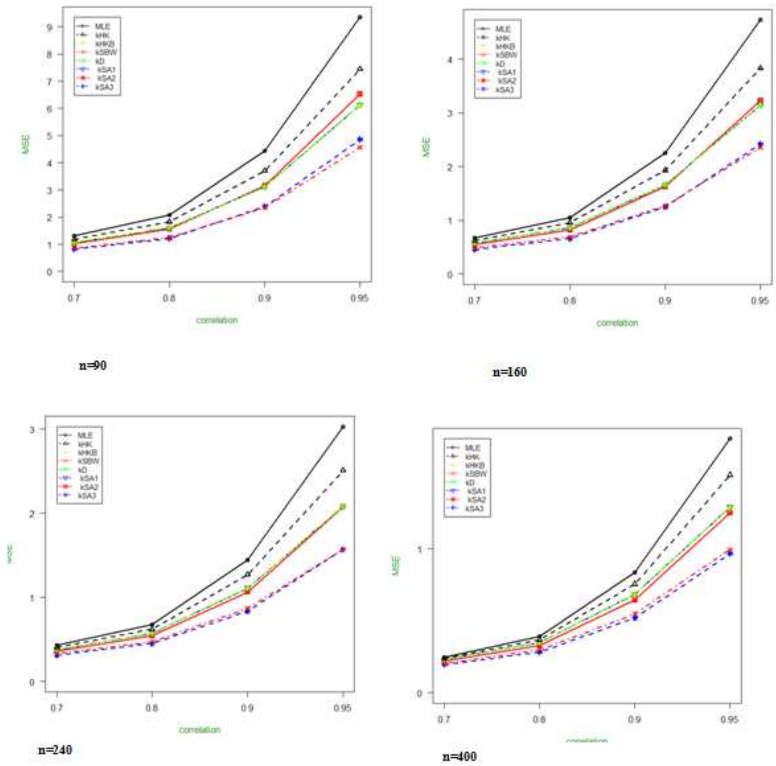

Figure 6: The AMSE of the ML and Different Ridge Estimators, For P=4, N=90, 160, 240 and 400 with Different Correlation

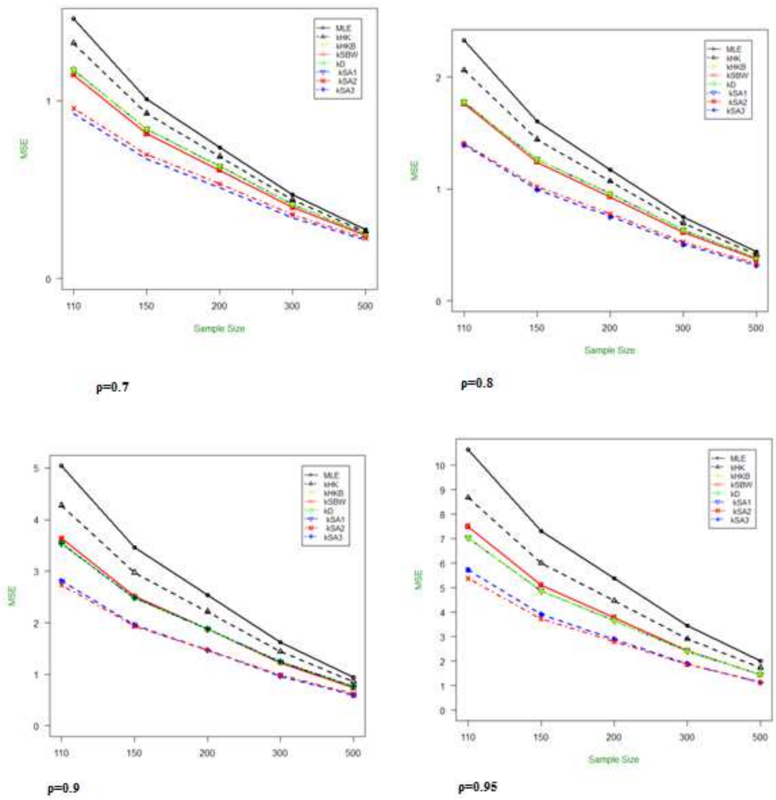

Figure 7: The AMSE of the ML and Different Ridge Estimators, For P=5, P=0.70, 0.80, 0.90 and 0.95 with Different Sample Size

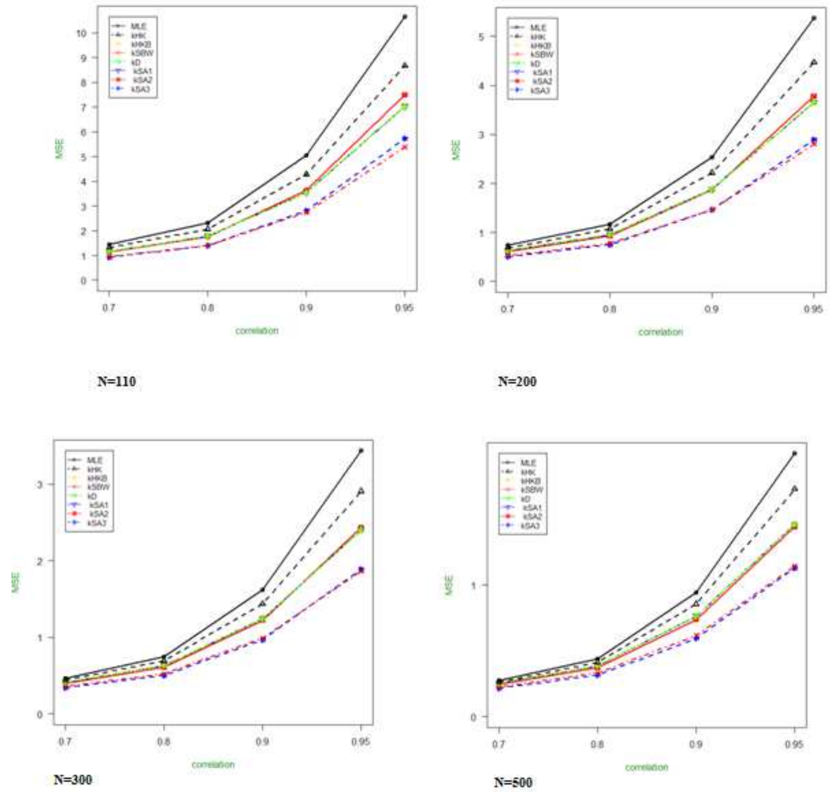

Figure 8: The AMSE of the ML and Different Ridge Estimator, For P=5, N=110, 200, 300 and 500 with Different Correlation

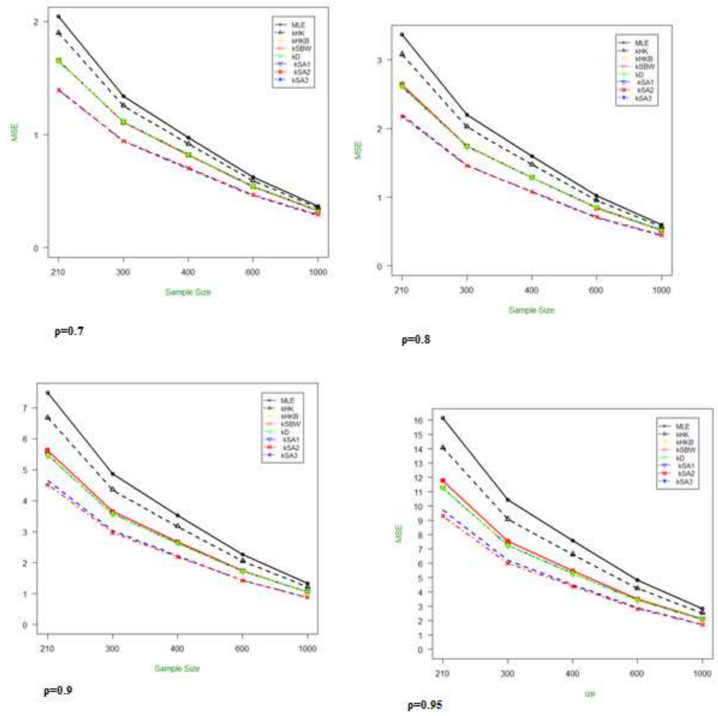

Figure 9: The AMSE of the ML and Different Ridge Estimators, For P=10, P=0.70, 0.80, 0.90 and 0.95 with Different Sample Size

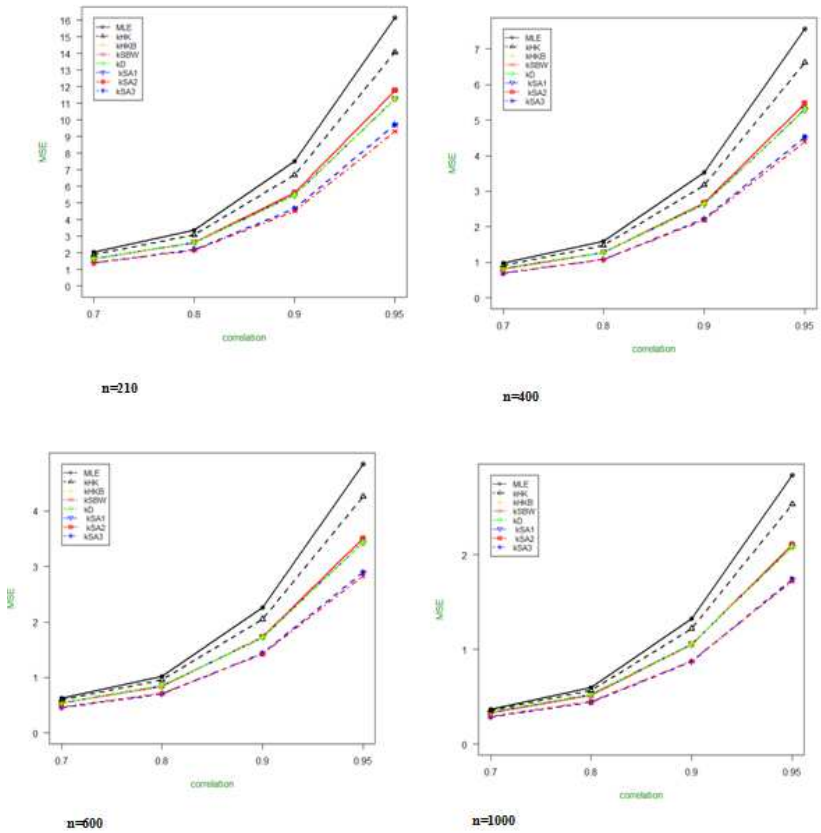

Figure 10: The AMSE of the ML and Different Ridge Estimators, For P=10, N=210, 400, 600 and 1000 with Different Correlation